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Compliance Regulation Strategies and Performance Evaluation

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Abstract

This deliverable focuses on the control of compliant actuators compliant robots developed within the AMARSI project, particularly the compliant humanoid CO-MAN developed at IIT and the compliant quadruped robot Cheetah developed at EPFL. The deliverable summarizes the outcome of tasks T2.5 and T2.6 related to the implementation and validation of different control strategies to regulate the motion of these soft body robots. These are activities include strategies to regulate the motion/impedance at the joint level as well as up at the center of mass (COM) and whole body level. The application of these controllers and their performance are validated on different tasks related to body stabilization against physical interactions and external push disturbances as well as in the generation of gaits based on kinematic motion primitives. To enhance the level of balancing capabilities impedance regulation explores optimal strategies for the selection of stiffness in particular joints while soft interactions with the environment are triggered to permit the humanoid to balance using additional contacts with the environment. This become possible thanks to the soft intrinsic and active compliant properties. All these works and experimental trials clearly demonstrated the benefits gained by the combination of active and intrinsic compliant properties.
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1 Robust Link Position Tracking Control for Multiple Compliant Joints Robot

In this work a practical and robust link position tracking control was developed for high degrees-of-freedom (DoFs) robotic system with compliant joints [?]. Passive compliance offers many advantages in terms of safe interaction with the environment and users. However, the compliance poses some challenges in particular tasks when the robot’s joints are under large gravitational forces or external disturbances. These scenarios include carrying a heavy load or manipulating heavy objects.

The proposed control provides fully decentralized link-tracking control without using a mathematical model, while general trajectory tracking controllers require the use of full-state feedback and knowledge of all the terms in the dynamics model. To achieve precise task-requirements and to reject the effect of external disturbances without the model, time-delay estimation (TDE) scheme is incorporated. The TDE uses minimum information of the plant to estimate and compensate the robot dynamics, that is, intentionally delayed information of acceleration and input torque.

A final control law for n-DoFs compliant joints robot to track desired link trajectory \( q_{Ld} \) can be summarized as

\[
\tau_m = \tau_{m(t-T)} - \hat{B}\dot{\theta} + \hat{B}(\dot{\theta} - \dot{\theta}) + \bar{K}_v(\ddot{\theta} - \ddot{\theta}),
\]

(1)

and

\[
\theta_i = \theta_{i(t-T)} - \bar{M}_K\dot{q}_{Li}(t-T) + \bar{M}_K(\dot{q}_{Li} - \dot{q}_{Li}) + K_D(\ddot{q}_{Li} + \ddot{q}_{Li}) + K_P(q_{Li} - q_{Li}),
\]

(2)

where \( \tau_m \in \mathbb{R}^n \) denotes a motor torque, subscript \( (t-T) \) denotes time-delayed signal in terms of a sampling period \( T \) (e.g., \( T = 1\text{ms} \)), \( \hat{B} \in \mathbb{R}^{n \times n} \) denotes nominal value of motor inertia, \( \theta, \dot{\theta}, \ddot{\theta} \in \mathbb{R}^n \) denotes angular position, velocity and acceleration of motors, \( K_v, K_p \in \mathbb{R}^n \) denote diagonal matrices, and \( \dot{\theta}_i, \ddot{\theta}_i \) are obtained from a numerical differentiation of \( \theta_i \); \( q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^n \) denote angular position, velocity and acceleration of links, \( \bar{M}_K \triangleq \hat{K}^{-1}\bar{M} \), where \( \hat{K} \) denotes the estimated joint stiffness matrix and \( \bar{M} \) denotes a diagonal matrix which may assume the nominal value of \( M \), and

\[
K_D = 2\omega_n\varsigma \cdot I_n \quad \text{and} \quad K_P = \omega_n^2 \cdot I_n,
\]

where \( \omega_n \) and \( \varsigma \) denote desired natural frequency and damping ratio of the system, respectively.

The closed-loop stability of the proposed control is proved as follows: if the sampling period \( T \) is sufficiently small and roots of \( [I_n - M^{-1}KM_K] \) and \( [I_n - B^{-1}B] \) reside in a unit circle, the closed-loop error dynamics of the overall system is asymptotically bounded within a very small positive value thus the overall system is bounded-input-bounded-output (BIBO) stable.

The proposed control scheme is validated and compared in a full body dynamic simulation of humanoid COMAN with 23 DoFs. First, the controller performance is assessed on the torso of COMAN as shown in Figure 1.1. In this simulation, COMAN Upper body, 8 DoFs is adopted using dynamic simulator Robotran. As the position responses
Figure 1.1: (a) The desired paths for eight joints of two arms; and (b) The desired trajectories of link angle for the tracking control: the desired position, velocity and acceleration.

Figure 1.2: The link position angle responses for the first two joints of left arm: the red solid line indicates the result of the proposed control; and the black dotted line indicates the desired trajectory.

are shown in Figure 1.2, the proposed control shows small tracking errors throughout the trajectory; the root-mean-squared (RMS) values of tracking errors of 0.1353 and 0.1390 deg are achieved in both fifth-order and sinusoidal trajectories for joint 1 and 2, respectively. Second, a whole body control is shown for carrying heavy loads during squatting motion, as shown in Figure 1.3. The proposed control scheme is compared with full motor PD control. In cases shown in Figures 1.4 (a) and (b), all 23 DoFs of the robot is controlled using the classical PD motor position control. The robot is holding a box which weighs 40 N, while performing a squat motion. The link deflections (particularly the ankles) get large enough, then the robot falls on the floor. It is observed when the robot is squatting with 50 N box, the robot sways forward and backward but manages to keep the balance. In both cases, the major joint deflection occurs on the ankle pitch joints which are under the largest gravitational load. In Figure 1.4 (c), the proposed control applied to all the 23 DoF joints. And the robot successfully keeps the stable posture and the box while performing the same squat motion. It can be seen that in the proposed method, the torso has achieved a more upright posture under the load disturbance. The accuracy of the proposed controller is shown by the improved ability of the robot to keep its center of mass within the support polygon which is due to the precise link tracking control under disturbance. This full body simulation of COMAN carrying a box while squatting was presented to show the effectiveness of the proposed
Figure 1.3: Snapshots of COMAN performing a squat motion while carrying a box with two arms.

Figure 1.4: Simulation results on squatting motion are illustrated: (a) PD motor position control is applied to all the joints with a 4 Kg mass; (b) PD motor position control is applied to all the joints with a 5 Kg mass; (c) the proposed control is applied to all DoFs with the 5Kg mass.

controller in disturbance rejection and helping the robot balance under considerable gravitational loads.
2 Upper limb compliant strategy exploiting external physical constraints for humanoid fall avoidance

This activity focused on a novel balance strategy which fundamentally differs from previous methods as it exploits contacts with the environment using the arms to prevent falling rather than only performing body posture control [1]. We believe that these upper limbs actions and the exploration of contacts with the environment is a vital characteristics that can significantly extend the balancing performance of humanoids under severe disturbances, making them eventually capable of operating in realistic workspaces.

The proposed method combines two control strategies, one for the upper body and one for the legs. The lower body runs an Inertial Measurement Unit (IMU) based balance controller that rejects small external disturbances and maintains the upper body upright posture during strong pushes. A second controller implemented on the upper body executes the arm motions necessary to establish contacts with the environment and support the balance of the robot. The configuration of the robot arms when the contacts with the environment surface are established, is derived through an arm posture optimization that attempts to align the arm Cartesian stiffness ellipsoid with a desired orientation maximizing the compliance in a direction perpendicular to the contact surface. Then, the impedance levels of the arms are dynamically regulated to improve the matching to the desired stiffness ellipsoid and to ensure that the reaction forces from the arm contacts are adequate to support the robot without inducing contact instabilities.

2.1 Lower Body Stabilizer

We briefly introduce here the lower body stabilizer which uses as input the attitude information measured from the IMU mounted on the pelvis of the robot. Based on these measurements and on the proprioception, a closed-loop control law estimates the posture of the lower body in the world frame, and then modifies this posture through inverse kinematics so as to achieve the three main goals of keeping a horizontal motion of the center of mass, slowing down the attitude variations of the trunk, and acting as a low-pass filter that rejects high-frequency vibrations at the pelvis. These three properties of the lower body stabilizer do not prevent the robot from falling, but they lead to a smoother and more predictable motion of the robot trunk, even just after a push. Thanks to the effect of the stabilizer, it becomes a reliable choice to use a threshold on the trunk pitch (see also previous section) to decide whether the robot is about to fall or not, and thus whether the arms should start a motion to reach the wall or not. The algorithmic and mathematical description of this stabilizer is out of the scope of the present paper.

2.2 Arm Reaction Controller

The goal of the arm reaction controller is to generate the arm configuration and initial impedance values required to establish stable contacts with the environment and assist the robot recovery against a disturbance. To achieve this the arm controller generates the arm posture and joint stiffnesses so as to achieve as close as possible a Cartesian stiffness ellipsoid with a desired orientation.
The Cartesian stiffness of the robot arm is described by the stiffness matrix:
\[ K_C = -\frac{\partial f}{\partial x} \]  
(3)
which represents the relation between the Cartesian wrench \( f \) and the Cartesian displacement \( x \). \( K_C \in \mathbb{R}^{m \times m} \) where \( m \) is the number of Cartesian degrees of freedom. It is also possible to define a stiffness matrix in joint space as the relation between torques \( \tau \) and displacement \( q \):
\[ K_q = -\frac{\partial \tau}{\partial q} \]  
(4)
\( K_q \in \mathbb{R}^{n \times n} \) where \( n \) is the number of joints. From Maxwell’s Reciprocal Theorem, all stiffness matrices are symmetric:
\[ K = K^T \]  
(5)
and in particular \( K_q \) is diagonal. The mapping from Cartesian stiffness \( K_C \) to joint stiffness \( K_q \) is given by:
\[ K_q = -\frac{\partial \tau}{\partial q} = \frac{\partial (J(q)^T K_C J(q) \Delta x)}{\partial q} = J(q)^T K_C J(q) - \frac{\partial J(q)}{\partial q} K_C \Delta x \]  
(6)
where \( J = \frac{\partial T(q)}{\partial q} \) is the manipulator Jacobian, \( T(q) \) is the forward kinematic of the manipulator. If the stiffness at equilibrium position is considered, then the second term in (6) disappear and it is reduced to:
\[ K_q = J(q)^T K_C J(q) \]  
(7)
The inverse problem of (7) gives the Cartesian stiffness matrix:
\[ K_C = J(q)^T K_q J(q)^T \]  
(8)
where \( J(q)^T = K_q^{-1} J(q)^T (J(q) K_q^{-1} J(q)^T)^{-1} \).
It is possible to formulate the inverse problem considering also the compliance matrices \( C_C \) and \( C_q \) defined as the inverses of the stiffness matrices:
\[ C_C = K_C^{-1} \]  
(9)
\[ C_q = K_q^{-1} \]  
(10)
Therefore, the same reasoning made in (6) and (7) and considering (10):
\[ C_C = J(q) C_q J(q)^T \]
\[ = J(q) K_q^{-1} J(q)^T \]  
(11)
Notice that in the Cartesian compliance matrix, no inversion of Jacobian is needed, which makes the computation lighter, and that is why we use it in our approach.
2.2.1 Arm posture optimization

Let us now consider the case of a 2R planar manipulator where \( \mathbf{q} = [q_1 \quad q_2]^T \) is the robot joint configuration and \( \mathbf{K}_q = \text{diag}(k_{q1}, k_{q2}) \) is the Joint stiffness matrix. In this case the translational Cartesian stiffness matrix and the joint stiffness matrix are both of size \( 2 \times 2 \). For a given arm configuration the Cartesian stiffness ellipsoid has its major axis and orientation along the largest eigenvalue of \( \mathbf{K}_C \) and the arctangent of the associated eigenvector. Let us now consider a desired ellipse characterized by \( \lambda_1, \lambda_2 \) and \( \theta \) representing the major axis, minor axis and orientation angle of the major axis respectively. The associated Cartesian stiffness matrix \( \mathbf{K}_{Cd} \) can be computed as:

\[
\mathbf{K}_{Cd} = \mathbf{R}_z(\theta) \mathbf{S}_{\lambda_1, \lambda_2} \mathbf{R}_z(\theta)^{-1}
\]

where \( \mathbf{R}_z(\theta) \) is the rotation matrix about \( z \) of \( \theta \) and \( \mathbf{S}_{\lambda_1, \lambda_2} \) is the diagonal matrix of eigenvalues.

To derive the arm configuration we then setup an optimization problem that regulates the arm posture to match the orientation of the desired stiffness ellipse. Considering (9) and by multiplying both sides by \( \mathbf{K}_C \):

\[
\mathbf{C}_C \mathbf{K}_C = \mathbf{I}
\]

leads in the following optimization problem:

\[
\min_{\mathbf{q}} \| \mathbf{C}_C \mathbf{K}_{Cd} - \mathbf{I} \|
\]

Since we want the arm end effector to be placed on the wall, an equality constraint is added, which for instance in the XZ planar case of Fig. ?? has the form:

\[
\mathbf{T}_x(\mathbf{q}) - m\mathbf{T}_z(\mathbf{q}) - p = 0
\]

where \( \mathbf{T}_p = [\mathbf{T}_x \quad \mathbf{T}_z] \) is the positional part of the forward kinematic \( \mathbf{T} \) and the wall is modelled in 2D as a line of equation \( x = mz + p \). The resulting optimization problem is therefore:

\[
\min_{\mathbf{q}} \| \mathbf{C}_C \mathbf{K}_{Cd} - \mathbf{I} \|
\]

s.t. \( \mathbf{T}_x(\mathbf{q}) - m\mathbf{T}_z(\mathbf{q}) - p = 0 \)

Additional inequality constraints are inserted to bound the joint position within certain limits:

\[
\mathbf{q}_{\text{min}} \leq \mathbf{q} \leq \mathbf{q}_{\text{max}}
\]

Taking into account the passive joint elasticity presented in some of the joints of the compliant humanoid COMAN the joint stiffness matrix was formulated by considering a series of two springs: a fixed passive \( k_{qf} \) and a variable active \( k_{qi} \) element (Fig.2.5).

Figure 2.5: Fixed Joint stiffness and variable Joint stiffness
Thus, the elements $k_{qj}$ of $K_q$ are computed as:

$$k_{qj} = \frac{k_{qfj}k_{qij}}{k_{qfj} + k_{qij}}$$  \hspace{1cm} (18)

The achievable range of joint stiffness is added to the optimization problem as a set of linear inequality constraints that bound the joint active stiffness $K_{qi} = [k_{qi1} \ k_{qi2}]$ by $K_{qimax}$:

$$0 \leq K_{qi} \leq K_{qimax}$$  \hspace{1cm} (19)

Therefore, the optimization problem including the variable stiffness is:

$$\min_{q, K_{qi}} \| C_q K_{Cd} - I \|$$

s.t. \hspace{0.5cm} $T_x(q) - mT_z(q) - p = 0$

$$q_{min} \leq q \leq q_{max}$$

$$0 \leq K_{qi} \leq K_{qimax}$$  \hspace{1cm} (20)

This is a non-linear optimization problem in $q = [q_1 \ q_2]^T$ with one non-linear equality constraint and two linear inequality constraints.

The optimization problem (20) tries to solve at the same time two tasks: an inverse kinematics task and the achievement of a desired Cartesian stiffness. Since, in our formulation, the position of the end-effector is treated as an equality constraint, this affects the achievable Cartesian stiffness as shown in Fig. 2.6.

In this work we consider a desired Cartesian stiffness ellipse with a large major axis parallel to the wall and a small minor axis so as to obtain a compliant behavior in the horizontal direction, but a stiff behavior if the end effector is moving vertically. Other desired ellipses can generate different behaviors and arm postures. The detailed discussion of other possible choices for the desired Cartesian stiffness is out of the scope of this paper.

Furthermore, the resulting optimization depends on the type and the weights of the norm used to calculate the optimization error. In our experiments we have chosen the Weighted Frobenious Norm, weighted with the desired Cartesian stiffness. Finally, since (20) is a non-linear optimization problem, there is no algorithm that can ensure that the global optimum will be found in a reasonable time, and the final solution depends highly on the given initial guess. We explored a possible solution to this problem by using a Particle Swarm Optimization step to move towards a configuration that can be used as good initial guess for the optimization (20).
2.3 Experiments

We have tested our strategy on the Compliant Humanoid Robot COMAN.

The experiments are made with COMAN standing in front of the wall at a distance $L \simeq 0.3[m]$, and the arms motion is triggered (i.e., the optimization) when the robot horizontal motion exceeds a threshold of $x^* \simeq 0.15[m]$. The passive value of the joints stiffness is $K_{qf} = \text{diag}(120.0, 120.0)[N m/\text{rad}]$. Horizontal pushes are applied to the robot. The desired translational Cartesian stiffness matrix is:

$$K_{Cd} = \begin{bmatrix} 10.0 & 0.0 \\ 0.0 & 120.0 \end{bmatrix} [N m]$$

The configuration obtained and resulting from the optimization is the one shown in Fig. 2.6a. The upper body of the COMAN is controlled using an impedance controller with an internal loop on the desired torques, while the waist and the lower body motions are controlled in position. The initial configuration of the robot upper body is set with low joint stiffnesses and as a result the arms stay down due to the gravity force. The stabilizer can easily handle small pushes but when the external force is large and the $x^*$ threshold is exceeded, the optimization is performed, and the arms move towards the optimized configuration with the optimized joint stiffnesses. This behaviour is shown in Fig. 2.7 from frames 1 to 6. Once COMAN is in contact with the wall, we continue to push it from both the shoulders at the same time. COMAN presents a compliant behavior when pushed and at the same time does not collapse on the obstacle. The single parameter $\alpha$ regulates the joint stiffnesses. A proper level of compliance is obtained by multiplying $\alpha$ by an experimentally tuned constant.

This behavior of the robot is shown in Fig. 2.7 from frames 7 to 11. To prove the robustness of our approach we also push the robot only on one shoulder, and the pushes are well absorbed. This is shown in Fig. 2.7 from frames 12 to 16. The control scheme is also robust to variations in the distance to the wall and to different configurations reached after the push.

Figure 2.7: Frames 1 to 6 show the reaction to a push. The robot moves its arms in order to achieve the configuration computed from the optimization step. Frames 7 to 11 show the behavior in case of an equally distributed push when the robot is in contact with the environment. Our strategy makes COMAN entirely compliant to the disturbances. Frames 12 to 16 show the same behaviour in case of a non-equally distributed push.
3 Ankle Optimal Compliance Balancing Stabilization for Humanoids

The aim of this work is to provide insights on how to generate effective ankle impedance profiles for the compliant humanoid balancing problem using optimal control strategies. It will be shown that for some range of impacts even a fixed value of passive stiffness in ankle can be adequate to provide optimal balancing performance. The optimal value of ankle impedance is derived analytically and numerically based on different cost functions in linear and nonlinear models to minimize the center of pressure and mass deviation. This optimal value is used in the ankle balancing strategy on COMAN robot and theoretical results are verified by experiment and simulation. Furthermore, the condition which ankle strategy fails is investigated [2, 3].

3.1 Compliant Ankle Strategy

3.1.1 Optimal Constant Compliance- Linear IP Model

The dynamic equation of motion\(-\tau + l \cdot \dot{F}_{ext} + mgsin(q) = (I_c + ml^2)\ddot{q}\) is linearized around \(q = 0\) assuming that the sagittal range of motion is confined to \(-10^\circ < q < 10^\circ\). Taking into account that \(\int_0^\epsilon F_{ext} dt = m\dot{q_0}\), where \(\dot{q_0}\) is the initial velocity, the impact force can be eliminated by considering an initial COM velocity as a result of the applied impact force. \(\epsilon\) is the ideal impact time which \(\rightarrow 0^+\). Considering \(u = \tau\) as the control input and \(x_1 = q\) and \(x_2 = \dot{q}\) as the system states, the equation can be written in state space form as follows:

\[
\begin{cases}
\dot{x} = Ax + Bu, \\
A = \begin{pmatrix} 0 & 1 \\ \frac{mg}{I} & 0 \end{pmatrix}, \\
B = \begin{pmatrix} 0 \\ -1 \end{pmatrix},
\end{cases}
\]

where \(I\) is the moment of inertia around pivot point and is \(I = I_c + ml^2\) by parallel axis theorem. \(K\) and \(D\) are the resultant torsional stiffness and damping respectively due to the active impedance controller and the passive elasticity which are in series. Therefore \(K = K_aK_p\) with the upper bound of the total stiffness being limited by the level of passive stiffness \(K_p\). In addition, in the linearized model the position of the COP \(x_{cop}\) can be approximated by \(\frac{1}{mg}\). The \(\ddot{z}\) term has minor effect on the \(x_{cop}\). Since \(x_{cop}\) inside the convex polygon, defined by the feet, is sufficient condition for whole body stability, an optimization cost function can be defined as:

\[
J_1 = \int_0^\infty x_{cop}^2 dt = \int_0^\infty \frac{\tau^2}{(mg)^2} dt = \int_0^\infty \frac{u^2}{(mg)^2} dt.
\]

Other options of cost function will be discussed later in this section with more generality. \(J_1\) reduces the problem to a standard minimum effort Linear Quadratic Regulator (LQR) problem resulting in the algebraic Riccati equations and control input which is linear feedback of the states.

\[
\begin{cases}
-Q - A^T K_x - K_x A + K_x BR^{-1} B^T K_r = 0, \\
u = -R^{-1} B^T K_x x = \frac{[m\dot{q}]^2}{f} (K_{r12}x_1 + K_{r22}x_2),
\end{cases}
\]

(23)
where $Q = 0$ and $R = \frac{1}{(mg)^2}$. $K_r$ is the Riccati matrix which is symmetric with three independent elements $K_{r11}$, $K_{r12}$ and $K_{r22}$. This matrix must be positive semidefinite to ensure a stable system. Solving the Riccati equations gives the optimal value of overall stiffness and damping which is the resultant of both active and passive impedance:

$$\begin{align*}
K_{r12} &= \frac{2l^2}{mg}, \\
K_{r22} &= \frac{2l^2}{mg} \sqrt{\frac{l}{mg}}.
\end{align*}$$

This feedback design can be implemented by placing a stiffness and damping in the ankle joint. This stiffness and damping could be a real physical compliance like a series elastic actuator (SEA) or a variable elastic actuator (VSA). In this case the ankle torque will be:

$$\tau = K(q - \theta) + D\dot{q},$$

where $\theta$ is the free length of spring and is considered constant in the upright postural position of $\theta = 0$. In analogy to torsional stiffness and damping equation (25), the optimal stiffness and damping will be derived from (24) with minimal $x_{cop}$ deviation:

$$\begin{align*}
K &= 2mgl, \\
D &= 2mg \sqrt{\frac{l}{mg}}.
\end{align*}$$

Therefore, the optimal stiffness and damping are constant as a function of the physical properties of the robot. The value of $K$ is twice the minimum value of stiffness, $mgl$, which ensures asymptotic stability for the robot, $mgl < K \leq K_p$. This level of optimum stiffness can be interpreted as follows. The normal pendulum creates torque around the pivot which originates from the gravity torque and is equal to $mgl$. To achieve the same stabilizing effect with the inverted pendulum a spring that generates equivalent torque of $mgl$ when perturbed from the equilibrium is required. Further to the stabilizing torque component, in the case of the inverted pendulum model the spring should generate additional torque to counterbalance the gravity torque that is also equal to $mgl$. In overall, a spring with stiffness of $2mgl$ as in (26) is required to stabilize the inverted pendulum similar to normal pendulum.

The natural frequency of the inverted pendulum system with the constant optimal value of stiffness (26) is $\omega_n = \sqrt{\frac{mgl}{I}}$. Accordingly $\frac{D}{T} = 2\xi \omega_n$, which results to a critically damped system with $\xi = 1$. The $\xi$ is damping ratio which is a common expression for second order vibration systems and should not be confused with other definitions of that in control theory. It’s value can be $0 \leq \xi < \infty$.

Simulation results for different initial COM velocities using the optimal stiffness and damping values given by (26) are presented in Fig. 3.8 and Fig. 3.9. In these two graphs the initial velocity of the COM varies from $0.2\text{rad/sec}$ to $0.7\text{rad/sec}$ with an increment step of $0.1\text{rad/sec}$. The position and velocity of ankle sagittal joint are plotted in Fig. 3.8 with larger impact forces resulting in increased initial velocities $\dot{q}_0$ for the $x_{com}$.

Since in static case, the projection of the center of mass on the ground should be located inside the foot polygon, introducing a penalty on the ankle deflection $q$ might also be considered. In this case the cost function can be formulated as follow:

$$J_2 = \int_0^\infty (x_{cop}^2 + \delta x_{com}^2) dt = \int_0^\infty \left( \frac{u^2}{(mg)^2} + \delta l^2 q^2 \right) dt,$$
where the $\delta$ denotes the center of mass deviation weight. The $x_{cop}$ has basically static and dynamic part. To understand this concept, center of pressure is derived as a function of $q$, $\dot{q}$ and $\ddot{q}$:

$$x_{cop} = \frac{mg\sin(q) - l\ddot{q}}{m(g - l\dot{q}\sin(q) - l\dot{q}^2\cos(q))}. \quad (28)$$

This equation can be linearized around the upright position assuming that $q\ddot{q} \approx 0$. The $x_{cop}$ can be substituted by the linearized form of (28). Since the $x_{com} = lq$, the (27) as a function of $q$ and $\ddot{q}$ is:

$$J_2 \approx \int_0^\infty \left( (1 + \delta)(lq)^2 + \left( \frac{l}{mg} \right)^2 \ddot{q}^2 \right) \, dt. \quad (29)$$

The COM projection to the ground, $x_{com}$, is the position component of the $x_{cop}$ which is $lq$. Therefore, $\delta > 0$ in (27) and (29) can be explained as giving more favor to the position deviation reduction of center of pressure. Also $\delta < 0$ will give more weight to the acceleration term of $J_2$. The $\delta$ can not be less than $-1$ which can cause a negative
optimal cost function. The \((27)\) will gives rise to the \(Q\) and \(R\) in the algebraic Riccati equations \((23)\) equal to:

\[
\begin{align*}
Q &= \begin{pmatrix} \delta l^2 & 0 \\ 0 & 0 \end{pmatrix}, \\
R &= \frac{1}{(mg)^2}.
\end{align*}
\]

Solving \((23)\) using \((30)\) yields:

\[
\begin{align*}
K_{r12} &= \frac{Il(1+\sqrt{1+\delta})}{mg}, \\
K_{r22} &= \frac{Il\sqrt{2(1+\delta)}}{mg}.
\end{align*}
\]

The general formula for the optimal stiffness, damping and damping ratio at this case by using \((23)\), \((25)\) and \((31)\) are:

\[
\begin{align*}
K &= mgl(1+\sqrt{1+\delta}), \\
D &= mg\sqrt{2l(1+\delta)}/mg, \\
\xi &= \frac{\sqrt{(2(1+\sqrt{1+\delta}))}}{2\sqrt{1+\delta}}.
\end{align*}
\]

This suggests that introducing the penalty on COM, \(\delta > 0\) which is equivalent to giving more favor to the position component of COP reduction in \((29)\) results to larger stiffness and damping than \(\delta = 0\) or \(\xi = 1\). Introducing a positive \(\delta\) implies to an under damped systems with \(\xi < 1\). It was predictable since giving more weight to the position part of COP in \((29)\) will get more freedom to the transient dynamics. The under damped systems has more dynamic transient freedom than over damped ones.

\[
\lim_{\delta \to \infty} \xi = \frac{\sqrt{2}}{2}. \quad \text{It is the minimum damping ratio which only consider the position component of } x_{\text{cop}} \text{ reduction and leads to a stable system. If only the } x_{\text{com}} \text{ deviation as a static approximation of } x_{\text{cop}} \text{ is considered then the limit of the center of mass will be:}
\]

\[
-sin^{-1}(d_2/l) < q < sin^{-1}(d_1/l) \quad (33)
\]

On the other hand, \(\delta < 0\) results to \(\xi > 1\) which implies to over damped systems and causes less stiffness and damping than the critical damped system \((\xi = 1)\). In these systems the proportion of transient dynamics is less than under damped systems. Therefore, giving more weight to acceleration of COP in \((29)\) minimize the dynamic movements. The pure static motion can be reached by killing the acceleration part of COP while \(\lim_{\delta \to -1} \xi = \infty\). In this case \(\lim_{\xi \to \infty} K = mgl\) which causes an asymptotic stability with very large settling time. The \(\xi = 1\) gives shortest settling time. Consider \(\delta = -1\) in \((29)\) then the cost is the minimization of the functional, \(\dot{q}^2\). The optimal control for this problem does not stabilize the system but it produces a zero cost when the initial velocity is zero and the initial position is not zero. In this case the system remains in static equilibrium since the controller produces torque to counteract the gravity.

For zero initial position and nonzero velocity, the optimal control will not produce a zero cost, although it will drive the system to zero velocity and acceleration, the position will be left at a nonzero value. Therefore, the controller will bring COM to a static equilibrium as time goes to infinity. However the closed loop system is still not stable because there is a zero eigenvalue.
3.1.2 Variable Optimal Compliance-Linear IP Model

So far, for small impacts, no limits placed on the control input and states since \( x_{\text{cop}} \) stays inside the feet polygon. However, this is not the case for large impact forces. Considering the foot size and boundary positions \( d_1 \) and \(-d_2\) and the fact that \( x_{\text{cop}} \) should be always inside the foot polygon the following limits for the ankle torque can be derived

\[
-d_2 < \frac{\tau}{mg} < d_1 \quad \text{or} \quad -mgd_2 < u < mgd_1.
\]

According to minimum Pontryagin principle, the optimal input \( u^* \), regardless of the optimization cost function \( (J_1 \text{ or } J_2) \) and any values of \( \delta \), is:

\[
\begin{align*}
  u^* &= mgd_1 & u > mgd_1 \\
  u^* &= u & -mgd_2 < u < mgd_1 \\
  u^* &= -mgd_2 & u < -mgd_2
\end{align*}
\]  

Taking into account the above imposed torque limits the maximum initial center of mass velocity (as a result of the external disturbance force) that can be supported without exceeding this torque limit is \( \max(\dot{q}_0) = \frac{mgd_1}{D} \) which is derived from (25) for zero \( q \) and \( \theta \). In this last expression if the optimal damping \( D \) from (26) is used, maximum initial velocity will be \( \max(\dot{q}_0) \approx 0.74 \text{ rad/sec} \). This initial velocity causes the maximum \( x_{\text{com}} \) deviation of \( q = 3.83^\circ \) which is in the limit (33). To note here that initial velocities larger than \( \max(\dot{q}_0) \) will simply force the \( x_{\text{cop}} \) to pass over the boundary position of the foot causing the robot to tip over.

The optimal controller is a state feedback scheme which can be implemented by the stiffness and damping gains \( K \) and \( D \) respectively as shown in (25). Therefore, it becomes clear that the initial jump of \( x_{\text{cop}} \) as seen in Fig. 3.8 is because of the torque generated by the damping component due to initial velocity \( \dot{q}_0 \) while the COM position at the beginning is zero the spring component does not produce any torque. By including the \( x_{\text{cop}} \) regulation in the optimal control problem the damping gain can be decreased during time to prevent exceeding the torque limits and keep the \( x_{\text{cop}} \) within the support polygon. As the natural frequency of the LIPM is \( \sqrt{\frac{K}{mgl}} \) therefore, stiffness and damping will be related to each other:

\[
D = 2\xi I \sqrt{\frac{K}{mgl}}
\]  

(35)

For high impacts it is necessary to regulate control gains online based on \( q \) and \( \dot{q} \) according to (34). From (35) first \( K \) is obtained which gives the second equation in (36) and then substituting \( K \) in (25) for \( \theta = 0 \) gives the first equation in (36)

\[
\begin{align*}
  \frac{\dot{q}}{4\xi^2} D^2 + \dot{\xi}D + mglq &= \tau_{\text{max}} \\
  K &= \frac{D^2}{4\xi^2} + mgl
\end{align*}
\]  

where \( \tau_{\text{max}} \) is equal to either \( mgd_1 \) or \(-mgd_2\) as in (34). For applying the control law (34) in practice the \( \xi \) is assumed to be constant before and after control signal reaches to its limit. When the input reaches to the extremum torques, impedance values using (36) will vary depending on \( q \) and \( \dot{q} \) and this is what called variable optimal compliance. The (36) creates a varying gains feedback control and it should be considered that the stability criteria for the constant feedback is not valid for these kinds of linear time varying systems. Since the first equation of (36) has two solutions always the closet to...
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Figure 3.10: Position, $q [\text{rad}]$ and velocity, $\dot{q} [\text{rad}/\text{s}]$ of ankle sagittal joint due to high impacts equivalent to initial velocities from 0.7 to 1.5 rad/sec by incremental of 0.1 rad/sec.

the previous one should be chosen. Equation (35) indicates that stiffness can not be set bellow $mg\ell$ and damping less than zero.

Fig. 3.10 to Fig. 3.12 present simulation results for initial velocities $\dot{q}_0$ from 0.7 to 1.5 rad/sec. The fixed optimal values for the stiffness and damping were computed from (34) for $\delta = 0$ resulting to $\xi = 1$. Regulation of the stiffness and damping was then implemented on the basis of (36). Fig. 3.11 shows that difference between $x_{\text{cop}}$ and $\tau/mg$ becomes bigger for large ankle angles however still $\tau/mg$ is a good approximation of $x_{\text{cop}}$.

Also this figure indicates that using (36) the $x_{\text{cop}}$ can be maintained within its maximum limits of $d_1 = 11.5 \text{ cm}$. Fig. 3.12 illustrates that the largest initial velocity that the system can cope is 1.52 rad/sec which leads to the minimum possible stiffness, $mg\ell$. The largest initial velocity the bigger deviation of stiffness and damping from its constant value of (26). For $\dot{q}_0 = 0.7 \text{ rad/sec}$ no reduction to the stiffness and damping is necessary. For the maximum initial velocity $\dot{q}_0 = 1.52 \text{ rad/sec}$, the ankle stiffness has to be decreased significantly resulting to the longest settling time. Fig. 3.10 shows that the maximum initial velocity can deviate ankle to 19.52° which is still in the limit of $-7.6^\circ < q < 19.77^\circ$.

3.1.3 Optimal Compliance- Nonlinear IP Model

As it is clear from Fig. 3.10 that initial velocities more than 1.2 rad/sec will lead the linear system to nonlinear regions. Also there are different kinds of external forces like smooth disturbances or constant pushes which will increase the ankle deflection making not valid the linear model. To find optimal stiffness and damping for a wide range of disturbances the more general nonlinear model can also be considered. The cost function in this case will be:

$$J_3 = \int_0^\infty x_{\text{cop}}^2 \text{d}t = \int_0^\infty \frac{\tau^2}{(mg + m\ddot{z})^2} \text{d}t,$$

(37)
Figure 3.11: $x_{cop}$ and $\tau/mg$ deviation of robot after impact due to initial velocities from 0.7 to 1.5 rad/sec by incremental of 0.1 rad/sec

Figure 3.12: Stiffness and damping variations for high impacts which cause $x_{cop}$ jump to border of foot polygon. Initial velocities from 0.7 to 1.5 rad/sec by incremental of 0.1 rad/sec

where $\ddot{z} = -l\left(\frac{mg}{I} \sin^2(x_1) - \frac{u}{I} \sin(x_1) + \frac{lF_{ext}}{I} \sin(x_1) + x_2^2 \cos(x_1)\right)$. The objective is to minimize the Hamiltonian function:

$$H = \frac{u^2}{(mg + m\ddot{z})^2} + \lambda_1 \dot{x}_1 + \lambda_2 \dot{x}_2,$$

(38)

where $\lambda_1, \lambda_2$ are costate variables. To minimize the Hamiltonian function for an unbounded optimal problem it is enough to solve following state and costate space differential equations:

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \frac{mg \sin(x_1)}{I} - \frac{u}{I} + \frac{lF_{ext}}{I} \\
\frac{\partial H}{\partial x_1} &= -\lambda_1 = \frac{\lambda_2 x_2^2}{(mg + m\ddot{z})^2} + \frac{mg \cos(x_1)}{I}, \\
\frac{\partial H}{\partial x_2} &= -\lambda_2 = \frac{4mlx_2^2 u \cos(x_1)}{(mg + m\ddot{z})^3} + \lambda_1,
\end{aligned}$$

(39)
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where \( A = 2ml\left( \frac{2m\sin(x_1)\sin(x_1)}{I} + \left( -\frac{u}{I} + \frac{F_{\text{ext}}}{I}\cos(x_1) - x_2^2\sin(x_1) \right) \right) \). The initial condition for solving these system of differential equations are \( x_1(0) = 0, x_2(0) = 0, \lambda_1(\infty) = 0, \lambda_2(\infty) = 0 \). The (39) can be solved by explicit Runge-Kutta (4,5) numerical method.

Equations (39) and (40) should be solved numerically as a set of simultaneous differential and algebraic equations to compute control input, \( u^* \) while the input is not saturated otherwise (34) should be considered.

\[
\frac{\partial H}{\partial u} = 0 = \frac{2u(mg+oz)}{u^2} - \frac{\lambda_2}{\lambda^2}. \tag{40}
\]

Fig. 3.13 shows the solution of (39) while the control limit (34) is considered. In this plot only the solution for the initial velocity of 1.3rad/sec is depicted and the difference between the linear and nonlinear model is shown. It indicates that even for the large impacts and deviation of \( x_{\text{com}} \) the linear model and the proposed method for the variable stiffness and damping could be enough for this type of external disturbance. It justifies that the (36) and (26) can be used instead of solving nonlinear equations for impact external forces.

However, the final solution for the control signal depends on the external force, \( F_{\text{ext}} \). Equation (26) can only be used in impact external forces. One might be interested to use the above mentioned optimal solutions of impact for other types of disturbances but it might not be the optimal solution. In practice the impact has a profile and is not ideal with an infinitesimal contact time. This suggests that the optimal ankle torque for other types of disturbances should be derived by solving (39). To note though that this requires \( F_{\text{ext}} \) to be available (by measurement or estimation) that is not always possible in practice.

3.2 Experiment

The COmpliant huMANoid COMAN is a full body humanoid with a size of five years old child. Its main characteristic is the passive compliant joints which are based on
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Figure 3.14: Optimal compliance stabilizer under impact experiment. The impact occurred in the first snapshot from the left.

Figure 3.15: Comparison of experimental results of constant stiffness and damping for optimal \((K = 2mgl \xi = 1 \delta = 0)\) and non optimal \((K = 1.27mgl \xi = 1.528)\) case and impact equivalent roughly to initial velocity of 0.7 rad/sec.

the series elastic actuation approach. The stiffness of the passive compliant joints are 520Nm/rad, 420Nm/rad, 390Nm/rad respectively for ankle, knee and hip. Two types of compliance regulators based on impedance and admittance schemes are available in the robot. Experiments were performed only on the lower body of the robot. To validate the discussed optimal stiffness and damping the following tests have done:

1. Stabilization by ankle compliance under small impacts.
2. Stabilization by ankle compliance under large impacts.

To apply a repeatable impact, a 5Kg weight suspended by a rope from a fixed frame is released from a specified height and collides with the wooden beam mounted on the waist of the robot. The \(x_{cop}\) and ankle joint \(q\) deviation due to external impact equivalent to initial velocity of 0.7 rad/sec is shown in Fig. 3.15. The value of ankle optimal impedance is constant, \(K = 110.0682 Nm/rad\) and \(D = 25.0964 Nm\cdot s/rad\) based on (26). It is clear from the plot that center of pressure approaches to the limit 11.5 cm and it decrease quickly after the impact for the optimal case. For instant a non optimal case is depicted in this plot. It is the over damped case \((K = 1.27mgl \xi = 1.528)\) and it is obvious that settling time is much larger than optimal one. The real impact in practice is not same as ideal impact with infinitesimal contact time. For
that reason the $x_{cop}$ rises in a very short time and then decreases quickly while in the ideal case the $x_{cop}$ jumps suddenly due to impact. Fig. 3.16 shows the large impact experiment which is equivalent to initial velocity of 1.0 rad/sec and $\xi = 1$. Then it decreases and the impedance is kept constant according to (26) until the robot reaches the equilibrium. Fig. 3.17 plots the online update of stiffness and damping. The difference between this figure and Fig. 3.12 is due to the difference between real and ideal impact, inaccuracy in the modeling and responding delay of controller. The fluctuation of $x_{cop}$ is due to the passive compliant elements in the waist of robot which are not controlled by admittance controller and cause fluctuation of the waist. This vibration can even be detected in Fig. 3.15 with $\dot{q}_0 = 0.7$ rad/sec. This vibrations will transfer to the feet and cause center of pressure fluctuation. However, the method is still effective and can maintain the balance stability of robot.
A Passivity Based Cartesian Admittance Control for Stabilizing the Compliant Humanoid COMAN

A generic stabilization framework which is applicable for both stiff and compliant humanoids was developed within this work activity [4, 5]. The control framework is applied to the passive compliant humanoid robot COMAN which is equipped with series elastic actuators. The stabilization control framework combines a compliance control and an intrinsic angular momentum modulation to achieve an agile and compliant interaction against external perturbations. The admittance based compliance control uses the force/torque sensing in both feet to regulate the active compliance for the position controlled system. The physical elasticity in the new full body COMAN is exploited for the reduction and absorption of the instantaneous impacts while the admittance control further dissipates the excessive elastic energy. The angular momentum controller reduces the overall inertia effect for providing more rapid reactions. Both the theoretical work and experimental validation were presented.

4.1 Principles of the Admittance Control

The admittance control uses F/T sensors mounted in feet to compute the torque generated by the ground reaction forces, and modulates the position references of the center of mass (COM) to replicate the compliant behavior as that of a spring-damper system. In this scheme, the admittance controller obeys the physics rule of “force $\rightarrow$ motion” causality, and all the forces/torques applied by the external forces, gravity and actuators cause the motion of the real robot. All these interactions subsequently cause the changes of the ground contact forces which are fed back to the admittance controller, so the active control action is able to correct the system behavior by generating a model-based reference trajectories.

4.1.1 Admittance based Compliance Control

The resultant torque applied on the pendulum model with respect to the coordinate $\sum q$ is the summation of the torque created by the GRF $\tau_{GRF}$, the gravity $\tau_g$ and the external disturbance $\tau_{dis}$. The equation to describe the dynamics without viscosity is given by:

$$I\ddot{q} = \tau_{dis} + \tau_{GRF} + \tau_g$$  \hspace{1cm} (41a)

$$\tau_{dis} = I\ddot{q} - \tau_{GRF} - \tau_g$$  \hspace{1cm} (41b)

$$\tau_{dis} \approx I\ddot{q} - \tau_{GRF} - mglq$$  \hspace{1cm} (41c)

where the gravitational torque $\tau_g = mgl\sin(q) \approx mglq$.

Let $Z_{ext}(s)$ be the output impedance observed from the external perturbation. In the frequency domain, the effective impedance $Z_{ext}(s)$ that appears at the output is

$$Z_{ext}(s) = \frac{\tau_{dis}(s)}{q(s)} = Is^2 - mgl - \frac{\tau_{GRF}(s)}{q(s)}$$  \hspace{1cm} (42)
Equation (42) indicates that the output impedance can be modified by the active control of $q$. The desired reference position for $q$ can be generated based on the torque feedback computed from the F/T sensors in feet. This scheme is termed as “Admittance Control” since the force and torque are the inputs of the controller and the position is the output. In such a case, the admittance control follows the “force→motion” causality while the real robot serves as the impedance.

$$q(s) = \frac{-\tau_{\text{GRF}}}{I s^2 - mgl - Z_d(s)}$$ \hspace{1cm} (43)

In the above equation $Z_d(s)$ denotes the desired impedance to be replicated.

$$Z_d(s) = I_d s^2 + B_d s + K_d$$ \hspace{1cm} (44)

where $K_d > 0$ is the stiffness and $B_d > 0$ is the viscous coefficient and $\zeta$ denotes the damping ratio. The impedance replication only modulates the stiffness and damping behavior, while the desired inertia term $I_d$ in the equation is set the same as the real inertia property of the physical system $I$.

Substituting $Z_d(s)$ from (44) into (43), yields

$$q(s) = \frac{-\tau_{\text{GRF}}}{mgl + B_d s + K_d}$$ \hspace{1cm} (45)

The equation in the time domain which also includes the equilibrium position is

$$mglq(t) + B_d \dot{q}(t) + K_d (q(t) - q_0) = -\tau_{\text{GRF}}$$ \hspace{1cm} (46)

Rewrite the equation in the discrete form using backward Euler method to obtain the derivative term, yields

$$mglq(i) + K_d(q(i) - q_0) + B_d \frac{q(i) - q(i - 1)}{T} = -\tau_{\text{GRF}}$$ \hspace{1cm} (47)

For an ideal position controlled system, the real angular position $q$ is identical to the reference $q_d$. Substitute $q_d$ into (47), we can derive the desired reference angle $q_d$ in the discrete form at the $i$th control step, given the feedback signal $\tau(i)$ and the control loop time $T$.

$$q_d(i) = \frac{K_d q_0 + \frac{B_d}{T} q_d(i - 1) - \tau_{\text{GRF}}(i)}{mgl + K_d + \frac{B_d}{T}}$$ \hspace{1cm} (48)

Equation (48) is the general equation for a 1-DOF stiff system to achieve the admittance control based on the mass pendulum model. Given the limited size of support area and the height of the COM, the pendulum’s rotation about the $x$ and $y$ axes are small.
4 A PASSIVITY BASED CARTESIAN ADMITTANCE CONTROL FOR STABILIZING THE COMPLIANT HUMAN ROBOT

enough to neglect the coupling effect due to the variation of inertia tensor. Applying the principle in (48), the formulas can be obtained in a decoupled form as follows.

\[
q_{dx}(i) = \frac{K_{dx}q_{x0} + \frac{B_{dx}}{m}q_{dx}(i-1) - \tau_x(i)}{mgl + K_{dx} + \frac{B_{dx}}{m}} \tag{49a}
\]

\[
q_{dy}(i) = \frac{K_{dy}q_{y0} + \frac{B_{dy}}{m}q_{dy}(i-1) - \tau_y(i)}{mgl + K_{dy} + \frac{B_{dy}}{m}} \tag{49b}
\]

The corresponding positions in the Cartesian space are

\[
\begin{align*}
\dot{x} &= z \cdot \tan(q_{dy}) \\
\dot{y} &= z \cdot \tan(-q_{dx}) \\
\dot{z} &= l \cdot \cos(\arctan(\sqrt{\tan^2(-q_{dx}) + \tan^2(q_{dy})}, 1))
\end{align*} \tag{50}
\]

The above equations consider an ideal stiff system, however, the references for the position controllers can also be obtained for a compliant system with the known torsion stiffness \(K_{sx}\) and \(K_{sy}\) around the \(x\) and \(y\) axes of the coordinate \(\Sigma_O\).

\[
q_{dx}^*(i) = q_{dx}(i) - \frac{\tau_x(i)}{K_{sx}} \tag{51a}
\]

\[
q_{dy}^*(i) = q_{dy}(i) - \frac{\tau_y(i)}{K_{sy}} \tag{51b}
\]

The physical damping \(B\) of the real system is omitted and considered relatively smaller compared to the desired damping \(B_d\) set in \(Z_d\). Substitute \(q_{dx}^*\) and \(q_{dy}^*\) into (50), the position references can be obtained for the system with physical compliance. For CO-MAN robot, we apply (49a) and (51b) for controlling the compliance in the lateral and sagittal planes respectively.

4.1.2 COM Based Inverse Kinematics

The geometric solution of the inverse kinematics problem is well presented in the literature. In this study, a link based inverse kinematics is developed regarding the hip as the base link and the ankle as the end effector. The COM based inverse kinematics is solved by the numerical iteration using the forward kinematics to compute COM and the link based inverse kinematics. The position and orientation of the ankle, the position of the COM and the orientation of the pelvis are the inputs for the COM based inverse kinematics. Fig. 4.18(a) presents the concept of COM position regulation by modifying the hip position given the fixed position and orientation of the feet and the fixed hip orientation.

\[
p_{i,\text{hip}}^h(k+1) = p_{i,\text{hip}}^h(k) + K_{3x3}(p_{i,\text{com}}^h(k) - \bar{p}_{i,\text{com}}^h(k)) \\
k = 0, 1, \cdots N \tag{52}
\]

In (52), \(K_{3x3}\) is a diagonal matrix with proportional gains, \(k\) is the number of iterations within each \(i^{th}\) control loop. Fig. 4.18(b) illustrates the numerical iteration where the previous hip position is the initial guess. The inner loop of link based inverse kinematics updates the modification of hip position in proportional to the COM error calculated by the forward kinematics. Typically, the number of loop \(N = 3\) results in a good precision. We set \(N = 5\) in the implemented code.
4 A PASSIVITY BASED CARTESIAN ADMITTANCE CONTROL FOR STABILIZING THE COMPLIANT HUMAN ROBOT

4.1.3 Angular Momentum Control

The admittance scheme only simulates variable stiffness and viscous damping as in (44) but keeps the inertia of the system unchanged. To deal with external perturbations, it is also interesting to understand how the system dynamics can be affected by the equivalent inertia in the impedance term. The proposed angular momentum controller exploits the redundant degree of freedom (DOF) of the upper body to modulate a zero intrinsic angular momentum for reducing the overall inertia effect. By using this algorithm, it was found that the system has a faster response to disturbances due to the reduced inertia.

In the coordinate $\sum_{O}$, the total angular momentum of the robot consists of two terms: one is the orbital term caused by the linear momentum of the overall COM about the pivot, which is unavoidable; the other is the intrinsic angular momentum caused by all the segments spinning around the overall COM. So zero intrinsic angular momentum means that the entire robot behaves as a point mass and the moment of inertia felt by the external load is only that of a point mass around the pivot $ml^2$. It is the minimum achievable inertia tensor in the impedance term (44). In this case, the net torque around the COM is zero, so the external torque only spins the COM around the pivot, as depicted in the right picture in Fig. 4.19(a). Without the control of the angular momentum, the exhibited inertia tensor would be coupled with $I_c$, as shown in the left figure in Fig. 4.19(a).

As shown in Fig. 4.19(b), by rotating the upper body in an opposite direction to that of the lower limbs, the angular momentum created by the upper body could cancel out the momentum created by the legs. Therefore, the net angular momentum around the COM is zero, $L_{x,y} = 0$. This creates a more agile reaction since the effective inertia is
smaller.

The full expression of $L_c$ can be calculated as follows. One can also refer to the formula in \[?\].

$$L_c = \sum_{i=0}^{n-1} \left[ (r_i - r_{com}) \times m_i (v_i - v_{com}) + R_i I_i R_i^T \omega_i \right]$$

(53)

where

$$r_{com} = \frac{\sum_{i=0}^{n-1} r_i \times m_i}{\sum_{i=0}^{n-1} m_i}, \quad v_{com} = \frac{\sum_{i=0}^{n-1} v_i \times m_i}{\sum_{i=0}^{n-1} m_i}$$

(54)

In (53), the first term is the moment of the linear momentum of the $i$th body segment with respect to the overall COM. The second term is the angular momentum of $i$th body segment around its local COM. $I_i$ is the inertia tensor of the $i$th segment in the segmental frame. $R_i$ is the rotational matrix of $i$th segment in $\sum O$. So $R_i I_i R_i^T$ is the inertia tensor in $\sum O$. $\omega_i$ is the angular velocity of the $i$th segment in $\sum O$. In (53), $r_{com}$ and $v_{com}$ are the position and velocity vector of the overall COM. $m_i$ and $m$ are the mass of $i$th segment and the whole robot respectively. So $L_c$ is the sum of the angular momentum of all the segments spinning around its overall COM.

The directly application of (53) demands significant computation time, and the derivatives of the position vectors cause a drifting issue. Therefore, an approximation is adapted for on-line computation instead of (53).

Define $\alpha$ and $\beta$ are the roll and pitch angles of the vector from the origin of $\sum O$ to the midpoint between two hips.

$$\begin{align*}
\alpha &= \text{atan2}(-y_{\text{hip}}, z_{\text{hip}}) \\
\beta &= \text{atan2}(x_{\text{hip}}, z_{\text{hip}})
\end{align*}$$

(55)

The following equations are used to approximate a zero intrinsic angular momentum for on-line computation.

$$\begin{align*}
I_{xx} \phi + J_{xx} \alpha &= 0 \\
I_{yy} \theta + J_{yy} \beta &= 0
\end{align*}$$

(56)
4 A PASSIVITY BASED CARTESIAN ADMITTANCE CONTROL FOR STABILIZING THE COMPLIANT HUMANOID COMAN

Table 1: Angular Momentum Controller (Pseudo code)

1. Compute rotational matrix of upper body using an initial guess
   \[ \theta_i = \theta_{i-1}, \varphi_i = \varphi_{i-1}, \psi_i = 0; \]
2. Update the rotational matrix to compute the waist joint angles;
3. Input waist joint angles and constant arm angles to the COM based inverse kinematics module;
4. Update the joint angles of legs, and update \( \theta_i \) and \( \varphi_i \) using (55) and (56);
4. Return to step 2 until number of iteration \( N \) exceeds.

Figure 4.20: Admittance scheme based on the FT sensor feedback

,where \( I \) and \( J \) are the inertia tensor of the upper body (including arms) and lower body (excluding feet) respectively. \( \varphi, \theta, \psi \) are the roll, pitch and yaw angles of the upper body. The pseudo code in Table 1 explains the integration of the angular momentum computation with the COM based inverse kinematics module.

Fig. 4.20 illustrates the overall control framework in which the updated motor position references are the final outputs sending to on board position controllers. Note that the torque applied by the actuators are the stress caused by the deformation of mechanical transmissions, the position controllers have no directly torque control. The torque exerted on the horizontal plane is computed by

\[
\tau_{x,y} = \tau_l + \tau_r + r_{f_l} \times f_l + r_{f_r} \times f_r
\]

(57)

4.2 Experiments

Three types of experiments were carried out to validate the proposed stabilizer.

1. Static, periodic and impulsive push disturbances, with the angular momentum control on and off respectively;
2. stabilization on a moving platform;
3. stabilization during bipedal walking.
In the first experiment, the stabilizer executed without the angular momentum control. Fig. 4.21 and 4.22 show both the torque $\tau_{x,y}$ measured from the feet and the position references and estimations of the COM. The COM estimations were computed from all the motor positions and link positions respectively, shown in green and blue lines. For the sagittal plane stabilization, the admittance control used (51b) as explained in Section 4.1. Therefore in Fig. 4.21, the reference determined by $q^*_{dy}$ has an offset which was in proportional to the torque load $\tau_y$. Fig. 4.22 shows that the real COM tracks the desired reference very well in the lateral plane due to the high stiffness. It also proves the feasibility to use (49a) for the lateral admittance control.

The comparison experiment switched on the angular momentum control in order to study the dynamical influence brought by the momentum controller. While applying the periodic disturbances, the experimenter persistently pushed the robot back and forth as fast as the robot could response. The COM position estimations in Fig. 4.23 and 4.24 demonstrated more rapid reactions by number of cycles versus time, which can also been observed in the accompanied video.

To quantitatively compare the difference, the spectrum of the torque measurement $\tau_{x,y}$ were analyzed. As shown in Fig. 4.25, without the momentum control, the maximum responsive frequencies were typically around $0.5\text{Hz}$ for $\tau_y$ and $1\text{Hz}$ for $\tau_x$, corresponding to sagittal and lateral responses. However, with the momentum modulation, the maximum responsive frequency of $\tau_x$ shifted up to $3\text{Hz}$, while the frequency band
of $\tau_y$ increased to $2 - 3Hz$. The angular momentum control is fairly effective for the lateral response because of the high physical stiffness. The effectiveness of sagittal responses is less because the springs acted as mechanical low-pass filter which reduced the control bandwidth of the actuation units.
The snapshots in Fig. 4.26, 4.27 and 4.28 show COMAN’s posture recovery from kick/push and inertia force disturbances produced by an accelerating/decelerating platform. An orange ruler was marked on the top of the snapshots to visualize the movement of the robot.

Figure 4.26: COMAN’s stabilization after sagittal disturbance (0.5s interval)

Figure 4.27: COMAN’s stabilization under lateral disturbance (0.5s interval)

Figure 4.28: COMAN’s stabilization on a moving platform (0.3s interval)

Figure 4.29: COMAN’s online stabilization while walking (0.5s interval)
Finally, the feasibility of implementing the proposed stabilization scheme for bipedal walking was explored. The vectors $r_f_l$ and $r_f_r$ in (57) were updated according to the gait parameter of a constant step width. The walking trajectories were generated by the COM state based pattern generator [?]. The stabilizer module computed the modifications of COM position $\Delta x, \Delta y, \Delta z$ using (50) and added to the desired COM position from the gait pattern generator. Fig. 4.29 displays the snapshots showing the stabilized walking.

To have a comparison study, in both walking experiments with and without the stabilizer, the COMAN robot walked 10 steps. An additional reason was that the robot was more likely to fall down without the stabilizer as the number of steps increased. The vertical ground reaction forces measured on feet are plotted from the experimental data. The robot weight is approximately 26 kg. As Fig. 4.30 shows, each foot had landing impacts and hence force fluctuated during the stance. When the robot started to stop after 12.2 s, the fluctuation of forces implied an oscillation introduced by the compliant structure. However, in the stabilized walking, the forces shown in Fig. 4.31 indicated neither severe foot landing impacts nor undesired oscillations after the gait stopped.
5 Compliant Attitude Control and Stepping Strategy for Balance Recovery with the Humanoid COMAN

This activity focused on the stepping strategy and practically addressed the problem of considering the stepping limitations of the robot when deciding to perform one step or more. Additionally, we use a stabilization control law and add atop of it a novel compliant attitude control scheme that slows down the dynamics of the robot, leading to smoother trajectories even in the presence of perturbations. Feedback from Inertia Measurement Units (IMUs) has been used extensively as a mean to control the balance of humanoid robots. Jenkins et al. introduce a method for inertial motion control inspired by how inverted pendulums such as Segway PTs or Golem Krang dynamically balance. They generalize this notion to humanoids, mimicking the action of a standard proportional-derivative (PD) motion controller that computes motor torques about each degree of freedom of an inverted pendulum, based on the differences between measured and desired angles and angular velocities. IMUs are used to measure the angular velocity of the robot, which makes it possible to control the dynamic balance of the humanoid robot with direct angular momentum feedback. In our approach, we use the IMU feedback to give a more compliant behavior to the robot, which already has an intrinsic compliance due to its series elastic actuators, and some additional active compliance due to the stabilizer. Our compliant attitude control algorithm uses as feedback only the measured attitude, not the angular velocities. It relies on one parameter that can be tuned to achieve a wide range of levels of compliance. Experiments on the lower body of the humanoid robot COMAN validated the combination of this compliant attitude control scheme with the stabilization control law and our stepping strategy. In the experimental results obtained, COMAN demonstrated a very robust behavior, making appropriate sequences of steps to keep its balance after strong pushes in any direction.

Figure 5.32: The global architecture of our control scheme. Reference trajectories are sent as triples of homogeneous matrices representing the rigid body motion of the waist, right and left foot in the world frame. Both controllers (the compliant attitude controller and the stabilizer) use feedback from the robot to update the values of these homogeneous matrices. They are then transformed into joint values through inverse kinematics, and a decentralized PID controller (one PID per joint) regulates the motor inputs.
5.1 Experimental Results

When pushed in any direction, the robot measures a disturbance and triggers appropriate steps Fig. 5.33. It can decide to perform two quick steps when one step only would not help much. Since the robot has just been pushed, it doesn’t necessarily track well the trajectory it tries to perform, but the compliance added by our attitude control algorithm ensures the smoothness of the motion. At the end of a step or sequence of two steps, the robot might still be unbalanced, for example if the initial push was strong. In that case, the robot can quickly decide to perform a new step or sequence of two steps. One very interesting property is that we can easily introduce artificial disturbances and use our balance recovery scheme as a robust gait generator. More precisely, an artificial disturbance of norm can be added to the measured disturbance, and it would result in the robot walking in the direction of the vector, but with the ability to withstand external perturbations and perform steps in any direction if needed. Besides, if a human operator wants to stop the walking motion of the robot, it is very easy to do so by pulling or pushing the robot in the direction opposite to its motion, with the real disturbance in this case naturally compensating the artificial one [6].

Figure 5.33: On the top: after a lateral push, the robot performs a sequence of two steps. On the bottom: after a frontal push, the robot decides to perform a single step backwards. However, just after this step the balance is not fully recovered, and the robot measures a disturbance indicating that it might be about to fall. As a result, it decides to perform a step backwards one more time, at the end of which the balance is fully recovered.
Humans, if compared to other animals, are able to accomplish a wider number of different tasks. In many cases, the motion produced is generated in real time to achieve a goal that has never been faced before. This flexibility is possible thanks to 1) their physical structure, that is the result of millennia of evolution, and to 2) their proficient neuromotor capabilities. This second point is fundamental to control a complex system like the human body.

The work on the kinematic Motion Primitives (kMPs) \[7, 8, 9\] was an analysis aimed at understanding how humans control the complex motion of their whole body, and reproduce the human skills on the COmpliant huMANoid (COMAN) robot.

![Figure 6.34: The test scenario with a human subject walking on a treadmill at different motions.](image)

The first part of the project focused on locomotion. Five subjects were asked to perform a set of walking and running trials, both unconstrained (free arms motion) and constrained (holding an empty box, or a 5 kg load with both hands). The whole body motion of the subjects was recorded with a Vicon motion capture system at 250 Hz. The 34 joint trajectories were obtained from the cartesian trajectories of the 39 passive markers placed on their body \[6.34\]. These joint trajectories were the basis of the analysis performed.

A Principal Component Analysis (PCA) was applied on each of the trajectories. What followed was the comparison between the main components returned by the PCA. It was noticed that the first 5 components, that together explained the 99% of joint trajectories variance, remained invariant among the different subjects \[6.35\] for different velocities of walking or running, or even other imposed constraints.

A statistical analysis confirmed what observed: all the different trajectories captured are accurately described by a small set of invariant signals, that we called kinematic Motion Primitives (kMPs).

The work is further developed to consider reaching, and two kMPs are identified for this class of motions. The kMPs extracted from both discrete and periodic motions can
The kinematic Motion Primitives (kMPs) and their application to generate a human-like walking with the COMAN robot.

Figure 6.35: The kinematic Motion Primitives of locomotion.

be combined to produce a new set of kMPs that describes the complex motion that is the simultaneous execution of the source basic motions (e.g., reaching for a target with one hand while walking). It is interesting to notice that, from the kinematic point of view, the combined motion is neither the sequencing nor the simple superposition of the source motions.

From the kMPs of locomotion a human-like center of mass trajectory was reconstructed and scaled down to the dimensions of the COMAN robot. This information, combined with valid engineered feet trajectories, was sufficient to generate human-like joint trajectories for our robot.

COMAN could perform a stable, highly dynamic, human-like walking\cite{6_36} with knees straightening up to $5^\circ$, and a big vertical displacement of the COM, whose motion excited the springs in the actuators of COMAN more than the usual engineered walking trajectories do, exploiting then the intrinsic compliance of the robot to store and release energy at the proper phases of the gait.

Figure 6.36: Snapshots of the COMAN robot walking with trajectories generated by reconstruction from kMPs.
7 Walking in the Resonance with the COMAN Robot

The benefits of compliance in terms of safety (for the robot, to reduce damages, and for the people around the robot) are known and have been studied exhaustively. But a compliant actuation system can also be exploited to improve energy efficiency. This also is theoretically known, and has been shown on simple robotic systems. Aim of the analysis presented in this section is to prove that this is valid also with a complex compliant system as the COMAN robot is.

7.0.1 Analysis of the Overall Energy Consumption

The features observed on the kMPs-based walk of the COMAN robot, described in Section 3.5, introduce a wide deflection of the springs in the actuators. For this reason this gait fits well for studying the relation between gait frequency and energy consumption.

![Energy consumption graphs](image.png)

Figure 7.37: Energy consumption at different gait frequencies a) to walk for 1 s, and b) to walk 1 m distance

The joint trajectories of the kMPs-based walking were scaled to obtain gait periods (frequencies) in a range from 0.8 s (1.25 Hz) to 2.0 s (0.5 Hz). The robot was able to walk at all the frequencies tested, and the overall energy consumption was recorded. Figure 7.37 shows the results collected in terms of energy per second [10][11].

The energy consumption per second grows as the motion becomes faster, and this was expected. But the distance covered by the robot walking at a lower frequency (and consequently more slowly) is minor as well. For this reason a more interesting information extracted is how much energy the robot requires to cover the same distance at different gait frequencies. Figure 7.37 shows that when the robot is walking at higher frequencies the energy consumption reduces significantly. According to the data reported, when the robot walks at a frequency that is in the neighborhood of one of the main resonance frequencies of the mechanism (0.8 Hz to 1.2 Hz) the energy required is about the 70% of the energy necessary to cover the same distance at lower frequencies. To verify whether this achievement was actually obtained by exploitation of the resonance, the contribution of the springs to the motion was analyzed in detail.
7.0.2 The Contribution of the Springs to the Motion

It has been proven that the energetic performance of the robot varies substantially as the gait frequency changes. This subsection aims to show that the intrinsic compliance of the actuators is in fact exploited when the robot walks in the resonance. The analysis performed shows that the springs are responsible of these performance variation: initially the overall contribution of the springs to the motion will be reported, down to the detail of the work done by the single springs. Figure 7.38a shows the total work done by the springs in a gait cycle.

![Figure 7.38: a) Total work done by the springs at different frequencies, and b) ratio between spring contribution and motor energy consumption](image)

The mechanical work done by the springs is considered as positive when it is contributing positively to track the joint reference trajectories, and as negative when the force generated by the springs goes in the opposite direction with respect to the desired trajectories. The contribution of each spring was integrated over one cycle, and the overall contribution of the springs is the sum of the contributions of the single springs. The total work of the springs is the difference in terms of energy consumption between the work done by the motors of the compliant robot, and the work that the motors of an equivalent stiff robot would have had to do to follow the same trajectories. This experiment verifies on a multi-dof compliant robot what often hypothesized in the literature: energy efficiency can be improved by exploiting the compliance of a robot. The total work of the springs can both be positive or negative: when the gait period is 1.4, for instance, the overall contribution of the springs is negative. In this case the energy consumption of the compliant robot is higher than it would have been if the robot was stiff. The best performance, instead, was measured when the robot was walking in the resonance.

![Figure 7.39: Comparison of energy efficiency between a stiff and a compliant robot](image)

Figure 7.39 extends the analysis on energy efficiency with a focus on the contribution of the single springs. A first observation is that there is no symmetry between left and right leg: this is because the reference joint trajectories are derived from the kMPs of
7  WALKING IN THE RESONANCE WITH THE COMAN ROBOT

Figure 7.39: Work done by the single springs at different frequencies

Figure 7.40: Detail on the positive and negative work done by the single springs at different frequencies

a human subject, and the original human gait was not symmetric. These graphs also reveal that the work of the springs, whether positive or negative, is major in the knees than in the ankles.
To understand better the behavior of the springs, in Figure 7.40 both the positive (blue line) and the negative (red line) contribution of the springs at the different frequencies are explicitly shown. The total mechanical work done by the springs corresponds to the dotted line above all the others, and is the sum of positive and negative work, without caring if it is helping the motion or not. When the robot walks in the resonance the springs are excited more, and both the positive and the negative work (in the knees) increase. The difference between the two, though, shows that the amount of positive work is major than the negative work around the resonance frequency (0.8 Hz to 1.2 Hz). This information is represented by the dotted line between the blue and the red lines, and correspond to what already reported in Figure 7.39.
8 The kinematic Motion Primitives (kMPs) and their Application to Generate a Horse-like Walking with the Cheetah-Cub Robot

Motivated by the good results of the application of human kMPs to generate a human-like walking, an extension to achieve a biologically-inspired quadrupedal locomotion was investigated. The research described in this Section aimed to identify the kMPs of horse locomotion, and to use them to directly transfer the biological features of horse gaits to a quadruped robot. The compliant quadruped robot Cheetah-Cub was used to test the horse-like trajectories synthesized. Subsequently, the trajectories were scaled in frequency to evaluate the performance of locomotion at different gait frequencies. Finally, a gait transition strategy is proposed. Developing an effective gait transition is fundamental for achieving a good performance in quadrupedal locomotion. Walk to trot and trot to walk transitions generated according to the proposed strategy were successfully tested on the Cheetah-Cub robot [12].

Figure 8.41: The source data (visualization at 12Hz) of the horse a) walking, b) trotting, and c) galloping on a treadmill.

The source data used in these experiments were purchased from the U.K. based commercial company Kinetic Impulse. These data consist of the joint trajectories of a horse (Figure 8.41) performing a walking gait, a trotting gait, and a galloping gait on a treadmill (horse dimensions: hips to the ground at rest position: 1.472m; hips to shoulders distance: 1.197m; right hip to left hip distance: 0.269m; right shoulder to left shoulder distance: 0.205m. These last two data refer to the distance between joints that are internal to the horse body, and are the first joints of the limbs from the spine). For each of the three gaits, three sequential gait cycles were arbitrarily selected, and the respective full-body joint angle trajectories (only those with a range of motion of at least 10° were considered) were averaged to reduce the peculiarity of the single cycle.

On these trajectories a Principal Component Analysis (PCA) was subsequently applied. For each of the three gaits the first four components were selected. In the case of the walk gait these components together accounted for 97% of the variance. Similar values were observed also in the case of the trot gait, and in the case of the gallop gait, with a cumulative percentage of variance accounted for of 96% and of 97%, respectively. These four components, normalized in time (from 0% to 100% of gait cycle) and amplitude (such that the maximum absolute value was 1) form the four kinematic Motion Primitives (kMPs) of walking, trotting, and galloping.

As it was in the case of human kMPs, a similarity among the kMPs extracted from the three gaits was noticed by observation. If in the walk and the trot gaits the order of the
kMPs remains the same, the first and the second kMPs of gallop have an inverted order with respect to the other two gaits. The third and the fourth kMPs, instead, maintain the same order in all the gaits analyzed. In Figure 8.42 the corresponding four kMPs from the walk, trot, and gallop gaits are superimposed to facilitate a comparison.

The similarity in terms of shape and phase observed by visualization in Figure 8.42 was confirmed by means of a statistical analysis that provided a quantification of the degree of similarity between gaits.

Cheetah-Cub, the compliant quadruped robot used for experimentation.

The similarity between kMPs of different horse gaits suggests that a unique set of four kMPs is at the basis of walk, trot, and gallop gaits. Hence, these kMPs can ideally be used to reconstruct any of the three gaits considered.

Since the values required to reconstruct a specific gait are not known, however, it was not possible to follow this procedure. The kMPs extracted from each gait, instead, were used to reconstruct the corresponding gait for the quadruped robot. PCA was applied on an enriched set of trajectories, that includes not only the joint trajectories, but also the four Cartesian foot trajectories with respect to a frame located in the middle of the spine. These trajectories are coupled with the joint trajectories, and it was verified that the kMPs extracted do not change when this information is added. In this way, though, the coefficients to reconstruct the Cartesian foot trajectories of the horse are known. The reconstructed foot trajectories are then proportionally scaled down according to the dimensions of the robot.

These trajectories had to be slightly modified to satisfy the mechanical constraints of the robot. They are first projected on the sagittal plane, since there is no adduction/abduction degree of freedom in the legs of the robot. Then their range of motion is further scaled down to ensure they do not exceed the joint limits. The range of motion of the legs of the robot, in fact, is smaller than it is for a horse, and this is mainly because the robot does not have a flexible spine. The reference positions for the motors of the robot are then derived from the foot trajectories with inverse kinematics. In the proposed approach the differences between the kinematics of the horse and the kinematics of the robot are not a limitation. What is maintained is the end-effector (foot) trajectory shape. This method does not require the robot to have the same kinematics as the horse. The robot could successfully perform a valid, stable locomotion in all three cases (Figure 8.43).
8.1 Analysis on the Effects of Gait Frequency Scaling

The quadruped robot could successfully perform stable walk, trot, and gallop gaits at their original frequency. The performance of gallop though was not satisfactory. High speed video footage indicates that this was caused by the missing spine movement: while horses and other mammalian quadrupeds extensively use their spine at high speed gaits, the robot has a stiff trunk. The method proposed matches the kinematic features of both legs and spine motion of the horse with the legs motion of the robot. The resulting trajectories required high torques to be tracked, and this led to massive inter-limb and contact forces that, consequently, caused major foot slippage. For this reason the experiments described from here on focused on the walk and trot gaits.

In the gaits reconstructed from kMPs, the decision had been taken to maintain the same gait frequency as the original one: \( f_{\text{walk}} = 0.97 \text{Hz} \) and \( f_{\text{trot}} = 1.59 \text{Hz} \), respectively (Table 2). The dimensions of the robot, though, are different from those of the horse whose gaits were used as source data. In the literature it is shown that gait frequency is inversely proportional to the dimensions of the animal itself. For this reason in the following experiments the gait frequencies of walk and trot gaits were increased systematically by a multiplication factor \( m = [1 : 0.5 : 3.5] \). In the case of the

<table>
<thead>
<tr>
<th>( m )</th>
<th>( f_{\text{walk}} )</th>
<th>( f_{\text{trot}} )</th>
<th>( v_{\text{walk}} )</th>
<th>( v_{\text{trot}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
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<td>1.59</td>
<td>0.13</td>
<td>0.28</td>
</tr>
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<td>0.24</td>
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<td>3.18</td>
<td>0.29</td>
<td>0.53</td>
</tr>
<tr>
<td>2.5</td>
<td>2.42</td>
<td>3.98</td>
<td>0.36</td>
<td>0.59</td>
</tr>
<tr>
<td>3.0</td>
<td>2.90</td>
<td>4.77</td>
<td>0.40</td>
<td>0.56</td>
</tr>
<tr>
<td>3.5</td>
<td>3.39</td>
<td>5.57</td>
<td>0.41</td>
<td>0.42</td>
</tr>
</tbody>
</table>
trot gait, this resulted in locomotion frequencies between \( f = 1.6Hz - 5.6Hz \). We expected a speed maximum roughly in the same frequency domain.

**Walk gait speed**

Results in Figure 8.44 indicate that the speed of the robot scaled almost linearly, with the base frequency being scaled up to a locomotion frequency of \( f = 2.42Hz \). The average robot speed at \( f = 2.42Hz \) was \( v = 0.35mps \). For two higher frequencies (2.9Hz and 3.39Hz), robot speed still increased. Plots in Figure 8.44 indicate that the maximum walking speed was reached close to the highest applied locomotion frequency of \( f = 3.39Hz \), with \( v = 0.43mps \), or roughly 2BL/s (body lengths per second).

![Figure 8.44: Speed versus frequency of three different gaits, run on the compliant quadruped robot platform. Walking speed ranged from 0.14mps to 0.42mps, with a speed-maximum at \( f = 2.9Hz \). Trotting speed ranged from 0.29mps to 0.62mps, with its speed maximum at \( f = 3.8Hz \). For gallop, only the base-frequency of \( f \approx 2Hz \) lead to a stable gait, with a speed of \( v = 0.2mps \).](image)

**Trot gait speed**

The base trot frequency applied (\( f = 1.59Hz \)) resulted in an average robot speed of \( v = 0.26mps \), or 1.2BL/s. The fastest trot gait frequency was found at 3.98Hz, with \( v = 0.59mps \), or 2.8BL/s. Speed of the robot increased mostly steadily until the maximum speed. The robot speed decreased for any higher control frequency, down to \( v = 0.42mps \) at \( f = 5.57Hz \).

### 8.2 Gait Transition in Quadrupedal Locomotion

The walk and trot gaits reconstructed from horse kMPs were the base gaits for the gait transition experiments conducted on the robot and reported in this section.

The foot trajectories of the four legs (Right Forelimb - RF, Left Forelimb - LF, Right Hindlimb - RH, Left Hindlimb - LH) for the walk and the trot gaits are resampled to
obtain 100 points per gait cycle. For each of the two gaits a "pivot" for the transition is identified:

\[
\begin{align*}
    \hat{i}, \hat{j} &= \min_{i,j} \left( d(RF_{Walk}(i), RF_{Trot}(j)) + d(LF_{Walk}(i), LF_{Trot}(j)) + d(RH_{Walk}(i), RH_{Trot}(j)) + d(LH_{Walk}(i), LH_{Trot}(j)) \right) \\
    &+ d(RF_{Walk}(i), RF_{Trot}(j)) + \\
    &+ d(LF_{Walk}(i), LF_{Trot}(j)) + \\
    &+ d(RH_{Walk}(i), RH_{Trot}(j)) + \\
    &+ d(LH_{Walk}(i), LH_{Trot}(j))
\end{align*}
\]

Pivots indicate those points in the walk and the trot gaits, respectively, such that the cumulative distance (among the four legs) between walk trajectory and trot trajectory is minimal.

In Figure 8.45 foot trajectories for walking and trotting are plotted together, and the pivot points of each gait are indicated with a dot. The transition period was set to take 3/4 of a gait cycle to be completed: 3/8 of trajectory before the pivot point, and 3/8 of trajectory after the pivot point of each gait are considered. The walk to trot transition trajectory is a weighted average of the walk trajectory and the trot trajectory, with the weight of trot growing linearly from 0 to 1, and the weight of walk decreasing linearly from 1 to 0.

Walk and trot also have different gait frequencies that result in a different sampling rate for the respective trajectories. The transition trajectory, hence, has a variable sampling rate: in the case of the walk to trot transition this rate changes linearly from the sampling rate of walk to the one of trot. Figure 8.46 shows the four foot trajectories during three different locomotion phases: a complete walk gait cycle, the transition from walk to trot, and a complete trot cycle. The samples are also shown. The reference motor positions derived from these data through inverse kinematics are sent to the robot at \( f = 50Hz \).

The foot trajectories for the trot to walk transition are defined in a similar manner, just with the weight of the walk increasing linearly from 0 to 1 and the one of trot decreasing linearly from 1 to 0, and the sampling rate going linearly from the sampling rate of trot.
Both types, walk/trot and trot/walk transitions were tested on the compliant quadruped robot. Experiments on scaling the gait in frequency were held as well (scaling factor $m = [1 : 0.5 : 3.5]$). The results of these experiments are reported in the next section.

**Experimental Results**

The gait transition characteristics of the robot are analyzed by measuring the range of pitch and roll movements for each gait after reaching steady state, and during the transition phase. In addition, instantaneous speed was recorded before, during, and after each transition. Roll and pitch angle phase plots are shown in Figure 8.48. Phase plots in Figure 8.48 are colored according to the commanded gait patterns, blue for trot gaits, orange for walking gait patterns, whilst green lines indicate transition times. The identical color coding was chosen for speed plots. Gait transitions were identified
and marked manually by placing start and end with the help of the speed plots, as a first rough approximation. Phase plots and characteristic shapes of walk and trot phase patterns were then used to separate more precisely transition times from steady gait locomotion. Qualitative plots are shown in Figure 8.48.

Frequency multiplication resulted typically in an increase in robot speed from walk to trot. All walk-trot and trot-walk gait transitions on flat ground were run successfully: the robot never stumbled nor fell. However, not all gait transitions worked equally well. One can see this by observing transition times and instantaneous transition robot speed (Figure 8.48). Typically, transitions went more smoothly with higher robot speed. In those cases transition happened more swiftly, and with less speed drop. Independently from the “direction” of the gait transition, stable patterns typically emerged after a
Table 3: Average transition times (in sec) for three transition strategies: Proposed method, No transition, and Not optimal pivot points. Average was taken over transition times of all multipliers. In average, the proposed method showed shortest transition times. Trot-walk transitions of the no-transition strategy had a tendency to produce non-stable walking gait patterns, with increasing pitch and roll angles (transition times only shown for stable runs).

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Walk-Trot</th>
<th>Trot-Walk</th>
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<tbody>
<tr>
<td>Proposed method</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>No transition</td>
<td>1.7</td>
<td>1.5</td>
</tr>
<tr>
<td>Not optimal pivot points</td>
<td>1.4</td>
<td>1.3</td>
</tr>
</tbody>
</table>

The gait transition took 26% longer for the “no transition” strategy, compared to the transition method proposed. In the second case (non-optimal pivot points), the transition time was 9% longer. This indicates that the choice of good pivot points improves the quality of gait transition (Table 3).
9 Kinematic primitives for walking and trotting gaits of a quadruped robot with compliant legs

In this work we reported on the interplay between a modular, feed-forward locomotion controller, and the mechanical entrainment of a quadruped, self-stably walking and trotting compliant legged robot [13]. We measured the complexity of the feed-forward controller, and the complexity of the resulting leg kinematics through the number of basic patterns accounting for a certain variance of kinematic data from in-air leg motions and on-ground locomotion, respectively. We implemented lateral sequence walk and trot gaits, and applied two different leg length control strategies. We found that the number of basic patterns from on-ground locomotion data matched those reported for animals; four basic patterns accounted for 95% of the variance. Three basic patterns accounted for 95% of the variance in lateral sequence walk and slower trot in-air experimental kinematic data, and two basic patterns accounted for faster trotting.

Figure 9.49: Schematic presentation of the in-air experiment (left), the on-ground experiment (middle), and a picture of the real robot (right). For the in-air experiment, feed-forward locomotion patterns (permutations of trot, lateral sequence walk, locomotion frequencies of 2.5 and 3.5 Hz, two different control pattern types) were sent to the robot, while the robot's body was mounted on a stand, with its legs swinging freely in-air. At the on-ground experiment, the quadruped robot walked and trotted freely on level-ground, with an average speed between 0.45 and 0.9 m/s. Identical motor control patterns were sent to both in-air and on-ground experiments, for each experiment type. In all experiments, resulting kinematic patterns (leg angle and leg length) were recorded. Change of complexity of kinematic primitives between in-air and on-ground derived from ground contact, and the robot's compliant leg design.

Because patterns were sent in a feed-forward manner, the measured complexity of in-air kinematic data represents the complexity of the feed-forward controller. This shows that already a simple, modular rhythm generator is sufficient for level-ground, feed-forward legged quadruped locomotion, for two different gaits walk and trot. It also shows that passive mechanical compliance enables an increase of kinematic complexity, leading to dynamic and self-stabilizing walk and trot locomotion. In the case of our quadruped legged robot, the complexity of the kinematic data increased at ground contact, through mechanical entrainment between the feed-forward controller and compliant, bio-inspired robot hardware. Here, the bio-inspired leg design supported the emergence of additional on-ground basic primitives, e.g., through passive leg compliance and leg segmentation. Animals show a much wider range of tools to adapt and modulate dynamic legged locomotion. Similar results between presented robot experiments and experiments with animals at level-ground locomotion indicate that modular, feed-forward, rhythmic pattern-based motor control in combination with compliant hardware are important components of animal neuro-control and biomechanics.
Figure 9.50: The percent variance accounting for the first 10 primitives, as a function of the number of primitives, for in-air stepping (red, dashed lines) and on-ground (black, solid lines) locomotion patterns. Horizontal, black, dashed lines indicate 80%, 90%, and 95% of the variance. In this article, we used a 95% of the variance interval (top, dashed, black, horizontal line). This results in between 2 and 4 primitives to account for 95% of the variance. The single peak (SP) trot gait at 3.5Hz (D) showed the largest change from in-air stepping to on-ground locomotion: 2 primitives accounted for more than 95% of the variance of in-air stepping data, and 4 primitives accounted for the same variance threshold, for on-ground locomotion. In the three other (AC) experimental setups 3 primitives accounted for at least 95% of the variance of kinematic in-air stepping data, and 4 primitives accounted for the same threshold of on-ground locomotion. (A) Walk, 2.5Hz, DP. (B) Trot, 3.5Hz, DP. (C) Trot, 2.5Hz, DP. (D) Trot, 3.5Hz, SP.
References


